Next Homework

Problems 7.5, 7.9, 7.11

Theoretical exercises: 7.4, 7.5

Chapter 5 continuous random variables

distribution function $F_X(a)$

density function $f_X(x)$

$$P\{a < X < b\} = \int_a^b f_X(x)\,dx = F_X(b) - F_X(a)$$

Expectation and variance

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$Var(x) = E[X^2] - (E[X])^2$$

Types of R.V.s

Uniform

Normal

Exponential

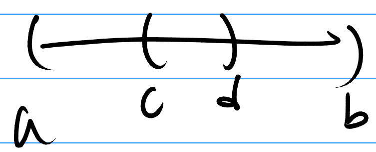
Gamma

What to use them for

Uniform - when different values are equally likely

$$f(x) = \begin{cases} \frac{1}{b-a} & a<x<b \\ 0 & \text{otherwise} \end{cases}$$

X uniform on (a,b)



$$\frac{d-c}{b-a} = P(c<X<d)$$

Normal with mean μ variance σ^2
std deviation σ

density and distribution are complicated transcendental function.

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2} \qquad \Phi(x)$$

→ Always try to transform into a standard normal RV. Z normal with $\mu=0$, $\sigma=1$

X has mean μ variance σ^2 $\iff$ $Z = \frac{X-\mu}{\sigma}$ has mean 0 variance 1

$$\Phi(a) = P\{Z \le a\}$$

normal

X = IQ test has mean $\mu = 100$, $\sigma = 15$

$130 = 100 + 2(15)$

$P\{X > 130\}$

$= P\{X > \mu + 2\sigma\} = P\{Z > 2\} = 1 - P\{Z < 2\}$

$= 1 - \Phi(2) = 1 - .9772 = .0228 \approx 2\%$

Can approximate Binomial distribution n, p

$\approx$ Normal with $\mu = np$, $\sigma^2 = np(1-p)$

if n is large $(\sigma^2 > 10)$

(Binomial to Poisson is for when p is small, n large)

Exponential $f_X(x) \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{other} \end{cases}$

Exponential models time to wait for something to happens (in a Poisson process)

$\rightarrow$ Memorylessness $P\{X > s+t \mid X > t\} = P\{X > s\}$

Gamma dist $\Leftrightarrow$ time to wait for α events to happen.

$$f_X(x) = \frac{\lambda e^{-\lambda x} (\lambda x)^{\alpha - 1}}{\Gamma(\alpha)} \quad \underline{\text{if } \alpha \text{ integer}} \quad \frac{\lambda e^{-\lambda x} (\lambda x)^{\alpha-1}}{(\alpha - 1)!}$$

Functin of RV

$g(x)$ function

increasing $g'(x) > 0$

thm $Y = g(X)$

$$f_Y(y) = f_X(g^{-1}(y))\,(g^{-1})'(y)$$

$$F_Y(y) = F_X(g^{-1}(y))$$

valid iff
y is in the range
of g

Chapter 6 multiple RV. joint density/mass functions

(X, Y) joint continuous

$$P\{a < X < b,\ c < Y < d\} = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

(Discrete) joint mass function $p(x,y) = P\{X = x, Y = y\}$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$p_X(x) = \sum_y p(x,y) \qquad p_Y(y) = \sum_x p(x,y)$$

X = roll one die   1 − 6

Y = roll another die   1−6

$p(x,y) = P\{X=x, Y=y\} = \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36}$ if $1 \le x \le 6$, $1 \le y \le 6$

$p_X(4) = p(4,1) + p(4,2) + p(4,3) + p(4,4) + p(4,5) + p(4,6)$

$= \sum_{y=1}^{6} p(4,y) = \underbrace{\frac{1}{36} + \cdots + \frac{1}{36}}_{6} = 6 \cdot \frac{1}{36} = \frac{1}{6}$

$$P\{X^2+Y^2 < 10\} = \iint_{\{x^2+y^2<10\}} f(x,y)\,dx\,dy$$

joint density function lets us express independence of R.V.s

$$f(x,y) = f_X(x)\,f_Y(y)$$

or $p(x,y) = p_X(x)\,p_Y(y)$

Conditional prob $P\{X=x \mid Y=y\} = \frac{p(x,y)}{p_Y(y)}$ discrete

Conditional prob. mass func. $= p_{X|Y}(x|y)$

Continuous

Conditional density func. $f(x|y) = \frac{f(x,y)}{f_Y(y)}$

X and Y indep if $\begin{cases} p_{X|Y}(x|y) = p_X(x) \\ f(x|y) = f_X(x) \end{cases}$

Take sums of independent R.V.s

discrete $P\{X+Y=a\} = \sum_y P\{X=a-y, Y=y\}$

$= \sum_y P\{X=a-y\} P\{Y=y\}$

$p_{X+Y}(a) = \sum_y p_X(a-y) p_Y(y)$

Continuous $f_{X+Y}(a) = \int_{-\infty}^{\infty} f_X(a-y) f_Y(y) \, dy$

- $\text{Normal}(\mu_1, \sigma_1^2) + \text{Normal}(\mu_2, \sigma_2^2) = \text{Normal}(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$
  if independent

- $\underset{\lambda_1}{\text{Poisson}} + \underset{\lambda_2}{\text{Poisson}} = \underset{\lambda_1+\lambda_2}{\text{Poisson}}$

$\boxed{\text{exponent + exponent} = \text{Gamma}}$