An Example of expectation

Exam 2 Stats

	Raw	Curved
Mean	78.1	85.1
1st Quartile	61.25	73.6
Median	81.5	87.4
3rd Quartile	96.75	97.8

$$\text{curved} = 100 - \left(\frac{15}{22}\right)(100 - \text{raw})$$

Continuous Random Variable X w/ probability density function $f(x)$

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$$

Example: Distributor sends parts to factory: shipping time is a random variable X w/ PDF $f(x)$.

If parts are early by s days, cost cs to store them.

If parts are late by s days, cost ks

due lost time.

When to send parts to minimize these costs?

t = # of days before needed that we send the parts

$t=10 \iff$ send parts 10 days before needed

$C_t(X)$ cost if we send parts t days before and it takes X days to arrive

Early : $(X-t) \leq 0$ $\quad C_t(X) = c(t-X)$

Late : $(X-t) \geq 0$ $\quad C_t(X) = k(X-t)$

$X = t \quad C_t(x) = 0$

X is a positive R.V. $\quad f(x) = 0$ for $x < 0$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

(analogous to $E[g(X)] = \sum_x g(x) P(X=x)$ for discrete)

$$E[C_t(X)] = \int_0^{\infty} C_t(x) f(x)\, dx$$

$$= \int_0^t c(t-x) f(x)\,dx + \int_t^\infty k(x-t) f(x)\,dx$$

$$= ct\int_0^t f(x)\,dx - c\int_0^t x f(x)\,dx + k\int_t^\infty x f(x)\,dx - kt\int_t^\infty f(x)\,dx$$

$\frac{d}{dt}$[this]

$$\frac{d}{dt} ct\int_0^t f(x)\,dx = c\underbrace{\int_0^t f(x)\,dx}_{=F(t)} + ct\underbrace{\frac{d}{dt}\int_0^t f(x)\,dx}_{=f(t)}$$

cumulative dist. func. $F(t)$

$$\frac{d}{dt}\int_t^\infty x f(x)\,dx = \frac{d}{dt} - \int_\infty^t x f(x)\,dx = -t f(t)$$

$$\frac{d}{dt}\left[t\int_t^\infty f(x)\,dx\right] = \int_t^\infty f(x)\,dx + t\frac{d}{dt}\int_t^\infty f(x)\,dx$$

$$= [1-F(t)] - t f(t)$$

$$\frac{d}{dt}(\text{stuff}) = cF(t) + ct f(t) - ct f(t)$$

$$- kt f(t) - k[1-F(t)] + kt f(t)$$

$$= cF(t) - k[1-F(t)]$$

$$= (c+k) F(t) - k$$

min $\quad 0 = \frac{d}{dt}(\text{stuff}) = (c+k)F(t) - k$

need $F(t) = \frac{k}{k+c}$

solution $t = F^{-1}\left(\frac{k}{k+c}\right)$

$F^{-1}(u)$ is inverse of cumulative distribution function

also known as quartile function of X.