Review for Exam 2

- 1 sheet of notes (2-sided) allowed
- Must be handwritten by you.

Next HW: Problems 5.2, 5.3, 5.4, 5.6, 5.7, 5.8

Random Variable = numerical function of the outcome of a probabilistic experiment.

Discrete RV = possible values can be indexed by integers
(or set of possible values is finite)

Possible values $X = x_1, x_2, x_3, \ldots$

For Discrete RV, have probability mass function (PMF)

$$p(x) = P\{X = x\}$$

This function contains most important information about X.

$1 \geq p(x) \geq 0 \qquad \sum_{\substack{x \text{ possible} \\ \text{values} \\ \text{of } X}} p(x) = 1$

$$P\{X \leq a\} = \sum_{x \leq a} p(x) = \sum_{x \leq a} P\{X = x\}$$

Expectation value = mean or average value

$$E[X] = \sum_{\substack{x \text{ possible} \\ \text{values}}} x\, p(x) = \sum_{x} x\, P\{X = x\}$$

Eg: In gambling examples X = winnings in a game

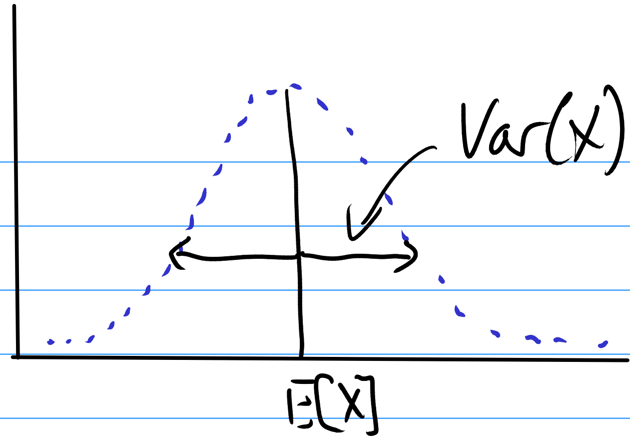
$E[X]$ = average winnings per game.

Variance $\quad \mathrm{Var}(X) = E\left[(X - E[X])^2\right]$

compute with this one $\rightarrow \quad = E[X^2] - (E[X])^2$

Variance measures how far X varies from its expectation value.

Small variance $\Rightarrow$ unlikely that X will deviate far from $E[X]$



Var(X) measures "width" of the distribution

Properties of E[X]

Alternative formula for expectation

Sample space S outcome $\sigma \in S$

X is a function of σ

$$E[X] = \sum_{\sigma \in S} X(\sigma) P\{\sigma\}$$

If $p(x) = P\{X=x\}$ is PMF of X

and $Y = g(X)$ where $g(x)$ is some function

$$E[Y] = \sum_{\substack{x \text{ possible} \\ \text{value} \\ \text{of } X}} g(x)\, p(x) = \sum_x g(x) P\{X=x\}$$

$$E[aX+b] = aE[X] + b \quad (a, b \text{ constants})$$

$$E[X^n] = \sum_x x^n P\{X=x\}$$

$$Var(X) = E[X^2] - (E[X])^2$$

$$= \sum_x x^2 P\{X=x\} - \left(\sum_x x P\{X=x\}\right)^2$$

If X and Y are two random variables

$$E[X+Y] = E[X] + E[Y]$$

Particular Random variables

Binomial
Poisson
Geometric
Negative binomial
Hypergeometric

all related to Bernoulli trials

sampling w/o replacement

Poisson process

Bernoulli trials (independent $P(\text{success}) = p$)

$X = \#$ of success in n trials

PMF $\quad P\{X=i\} = \binom{n}{i} p^i (1-p)^{n-i} \quad i = 0, \dots, n$

$E[X] = np$

$Var(X) = np(1-p)$

Binomial w/ params (n, p)

Geometric RV:

$Y = \#$ of trials to first success

$P\{Y=i\} = (1-p)^{i-1} p \quad i = 1, 2, 3, \dots$

$E[Y] = \frac{1}{p} \quad Var(Y) = \frac{1-p}{p^2}$

Negative Binomial

$Z = \#$ of trials for r successes

$P\{Z=n\} = \binom{n-1}{r-1} p^r (1-p)^{n-r} \quad n = r, r+1, \dots$

$E[Z] = \frac{r}{p} \quad Var(Z) = \frac{r(1-p)}{p^2}$

Poisson Process expected rate λ of events

$X(t)$ = # of event in interval of time of length t

$$P\{X(t)=k\} = e^{-\lambda t}\frac{(\lambda t)^k}{k!}$$

$$E[X(t)] = \lambda t \qquad Var(X(t)) = \lambda t$$

Or we could take λ to be the expected # of events.

The Poisson random variable with parameter λ

$$P\{X=k\} = e^{-\lambda}\frac{\lambda^k}{k!}$$

Binomial to Poisson approximation

If n is large, p is small, but np is moderate then Binomial RV w/ parameters (n,p)

$\approx$ Poisson RV w/ parameter $\lambda = np$

That is

$$P\{X=i\} = \binom{n}{i} p^i (1-p)^{n-i} \approx e^{-\lambda} \frac{\lambda^i}{i!}$$

(where $\lambda = np$)

Hypergeometric N balls m red
N−m blue

Select n balls w/o replacement

$X = \#$ of red in sample

$$P\{X=i\} = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}$$

$$E[X] = \frac{nm}{N}$$

Continuous RV X has probability density function $f(x)$

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$