Continuous Random Variables (§5.1)

Next Midterm 3/23

Material up to Lecture 22 (today)

- No book, No calculator
- Yes Notes
  - 1 US Letter (8.5" x 11") sheet
  - 2-sided
  - Handwritten by you (collaboration allowed)
  - No Xeroxing
  - Cram as much in as you want
  - No Magnifying glasses

⚠ Might see things like $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

Continuous Random Variables are just like discrete R.V. <u>except</u>

every sum becomes an <u>integral</u>

Continuous R.V. can take a continuous range of values

Eg. Possible values of X

could be $(0,1) = \{x \in \mathbb{R} \mid 0 < x < 1\}$

$[0,1] = \{x \mid 0 \leq x \leq 1\}$

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$(-\infty, \infty)$ = all real numbers

X = price of a stock

X = time that a machine works before breakdown

X = error in an experimental measurement.

How to describe probability associated with X?

[Recall for discrete R.V., we have probability mass function $p(i) = P(X=i)$

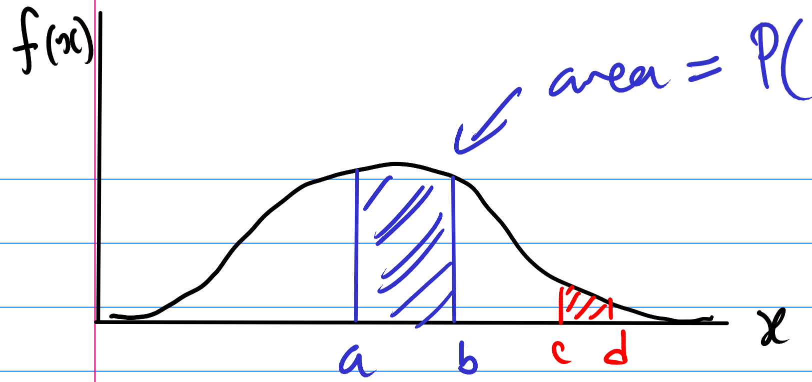
$$P(a \leq X \leq b) = \sum_{a \leq i \leq b} P(X=i)$$]

For continuous R.V. we have a function $f(x)$

such that $P(a \leq X \leq b) = \int_a^b f(x)\,dx$

$f(x)$ is called the <u>probability density function</u> of X

area $= P(a \le X \le b)$

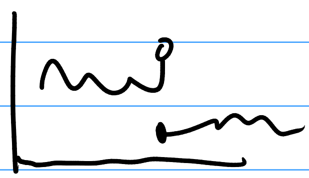


Because integral = area under curve

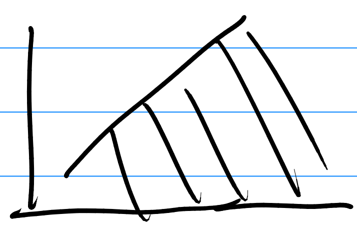
$$P(a \le X \le b) > P(c \le X \le d)$$

Basic properties of f

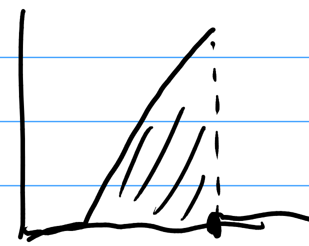
- $f(x) \ge 0$ everywhere
- $\int_{-\infty}^{\infty} f(x)\,dx = P(-\infty \le X \le \infty) = 1$
- $f(x)$ need not be a continuous function but it will usually be piecewise continuous



- $\int_{-\infty}^{\infty} f(x)\,dx$ exists $\Rightarrow$ $f(x) \to 0$ as $x \to \pm\infty$

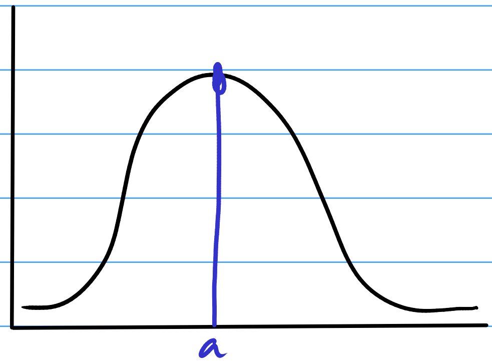


no

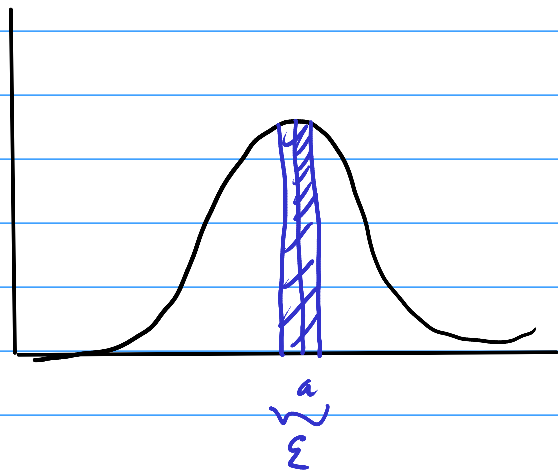


yes

$$P(X=a) = \int_a^a f(x)\,dx = 0 \neq f(a)$$



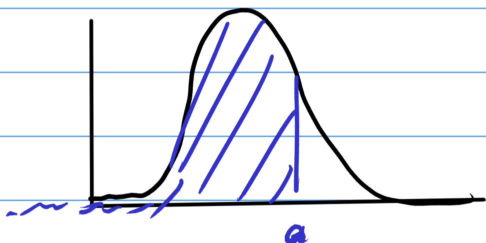
$$P\left(a-\frac{\varepsilon}{2} \le X \le a+\frac{\varepsilon}{2}\right) = \int_{a-\frac{\varepsilon}{2}}^{a+\frac{\varepsilon}{2}} f(x)\,dx \approx \text{width} \times \text{height}$$
$$= \varepsilon f(a)$$



Also still have Cumulative distribution function

[for discrete RV $F(a) = \sum_{i \le a} P(X=i)$]

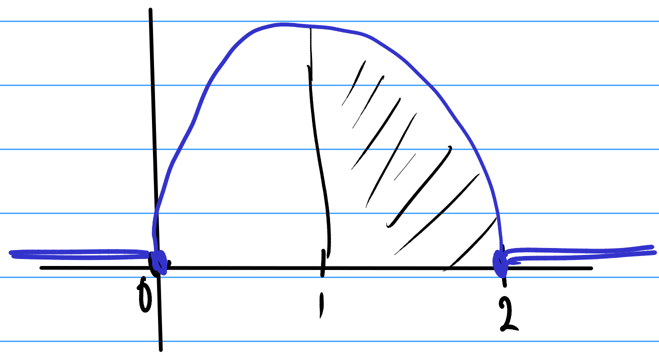
$$F(a) = P(X \le a) = \int_{-\infty}^{a} f(x)\,dx$$



$$\left.\begin{array}{l} P(a \le X \le b) \\ P(a \le X < b) \\ P(a < X \le b) \\ P(a < X < b) \end{array}\right\} \text{ all equal}$$

Example X continuous R.V. with density

$$f(x) = \begin{cases} C(4x-2x^2) & \text{if } 0<x<2 \\ 0 & \text{otherwise} \end{cases}$$



C is a normalization

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{2} C(4x-2x^2)\,dx + \int_{2}^{\infty} 0\,dx$$

$$= \int_{0}^{2} C(4x-2x^2)\,dx$$

$$= C\left[2x^2 - \frac{2x^3}{3}\right]_0^2 = C \cdot \frac{8}{3} \longrightarrow C = \frac{3}{8}$$

$$P(X>1) = \int_1^\infty f(x)\,dx = \int_1^2 f(x)\,dx \left(+\int_2^\infty 0\,dx\right)$$

$$\int_1^2 \frac{3}{8}(4x-2x^2)\,dx = \frac{1}{2}$$

Relationship between PDF (density) and CDF (cum dist.)

$$F(a) = \int_{-\infty}^{a} f(x)\,dx$$

$$\frac{d}{da}F(a) = \frac{d}{da}\int_{-\infty}^{a} f(x)\,dx \overset{FTC}{=} f(a)$$