# Discrete Random Variables

HW 6: Ch 4 Problems: 4.13, 4.18, 4.19, 4.20, 4.28

Theoretical: 4.2, 4.3, 4.4, 4.7, 4.8

Random Variable = function on the sample space.

$P(X = a)$

A Discrete Random Variable is one which can take on at most countably many values

<u>Ex</u> If X has finitely many possible values, then X is <u>discrete</u>

<u>Ex</u> If X has every integer as a possible value then X is discrete

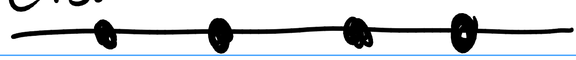
<u>Not discrete</u> X = amount of time it takes finish a race

possible values: $\{t : 0 \leq t < \infty\} = [0, \infty)$

all real #'s in this interval

Then X is a continuous random variable

discrete:



$X$

continuous

$X$

Thing about discrete R.V.: In general there are gaps between the possible values, and the possible values can be indexed by integers

possible values of $X = \{x_1, x_2, x_3, \dots\}$

PROBABILITY MASS FUNCTION of a discrete random variable $X$:

Let $a$ be a possible value of $X$

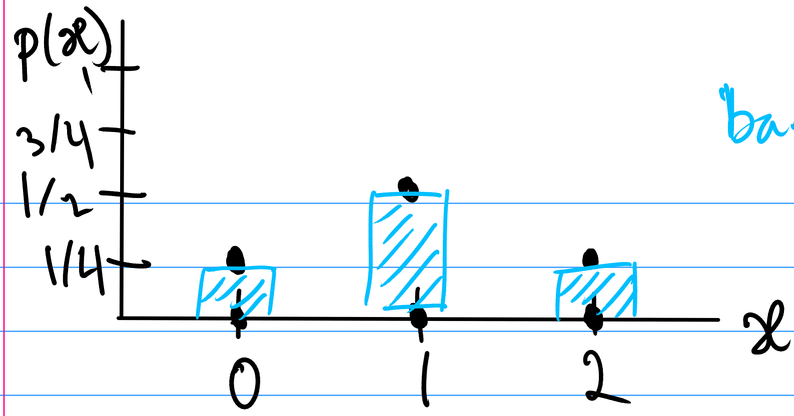
define $p(a) = P(X = a)$

$\underline{\text{Ex}}$ flip two coins $X = \#$ of heads

$$S = \{HH, HT, TH, TT\}$$
$X =$ 2, 1, 1, 0

possible values 0 1 2

$P(X=0) = \frac{1}{4}$ $\quad P(X=1) = \frac{1}{2}$ $\quad P(X=2) = \frac{1}{4}$




bars just make graph prettier,

possible values of X

and p(X) is zero at all other points

Basic properties of P.M.F.

possible values of $X = \{x_1, x_2, x_3, \ldots\}$

$1 \geq p(x_i) \geq 0$ (by Axiom 1)

$P(x) = 0$ if $x$ is not possible value

$$\sum_{i=1}^{\infty} p(x_i) = 1 \quad \left(\text{by Axiom } 3\right)$$

$$\sum_{i=1}^{\infty} p(x_i) = \sum_{i=1}^{\infty} P(X = x_i) = P\left(\bigcup_{i=1}^{\infty} \{X = x_i\}\right)$$

$$= P(S) = 1$$

Everything having to do with discrete R.V. is expressed it terms of (finite or infinite) sum

For continuous R.Vs, these sums are replaced by 'integral's.

For many questions about R.V.s, we don't care about sample space or the interpretation in terms of some experiment, we just care about the probability mass function of the R.V.

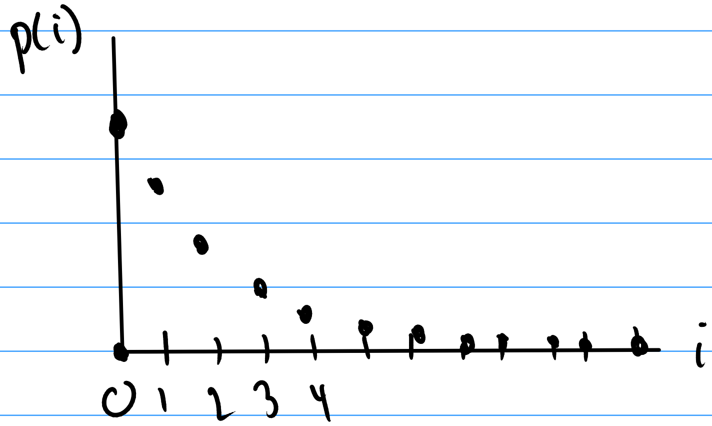
<u>Example</u> Some Random Variable $X$ has

P.M.F $\quad p(i) = c\lambda^i / i!$

Possible values $i = 0, 1, 2, \ldots$

$\lambda$ is a given constant

$c$ is an (as yet undetermined) normalization constant.



<u>Q</u> Relationship btwn $c$ & $\lambda$

Need

$$\sum_{i=0}^{\infty} p(i) = 1$$

$$1 = \sum_{i=0}^{\infty} p(i) = \sum_{i=0}^{\infty} c \frac{\lambda^i}{i!} = c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = c e^{\lambda}$$

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$ Taylor Series for the exponential function.

In other words $c = \frac{1}{e^{\lambda}} = e^{-\lambda}$

$$p(i) = e^{-\lambda} \lambda^i / i!$$

Compute probabilities using P.M.F.

$$P(X=0) = p(0) = e^{-\lambda} \frac{\lambda^0}{0!} = e^{-\lambda}$$

$$P(X=1) = p(1) = e^{-\lambda} \frac{\lambda^1}{1!} = \lambda e^{-\lambda}$$

$$P(X=2) = p(2) = e^{-\lambda} \frac{\lambda^2}{2!} = \lambda^2 e^{-\lambda}/2$$

$$P(X>2) = 1 - P(X=0) - P(X=1) - P(X=2)$$

$$= 1 - e^{-\lambda} - \lambda e^{-\lambda} - \lambda^2 e^{-\lambda}/2$$

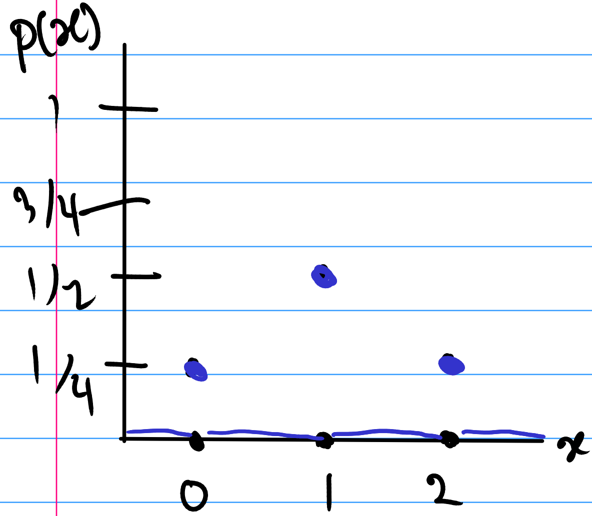
Another answer: $P(X>2) = \sum_{i=3}^{\infty} p(i)$

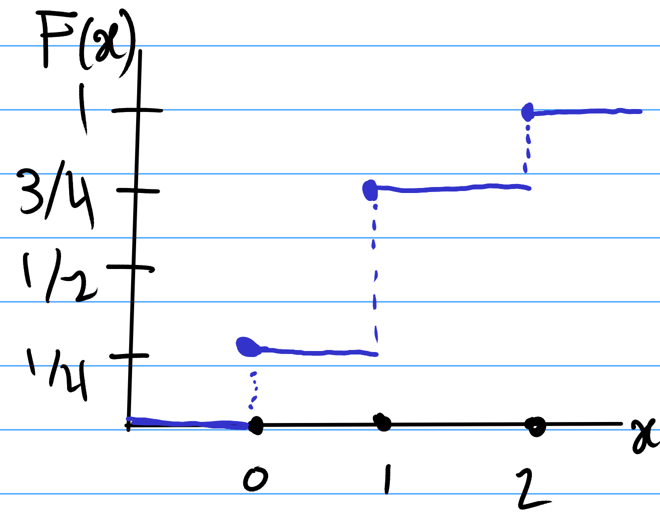
To any Random Variable $X$ can associate another functi

call Cumulative distribution function

$$F(x) = P(X \leq x)$$

$$F(x) = \sum_{a \leq x} p(a)$$


p(x)
1
3/4
1/2
1/4
0
1
2
x


F(x)
1
3/4
1/2
1/4
0
1
2
x