Random Variables

Exam Stats in terms of Raw Scores

1st Q	48
Median	67
3rd Q	86
Max	100

$C = \frac{1}{2}(R - 100) + 100$

Range is compressed to 50-100

Random Variable ← This idea for rest of course.

Interested in a function of the outcome of some probabilistic process.

Eg. 1) Roll two dice X = sum of dice

2) Flip 3 coins X = # of heads

3) Give a student a midterm X = score

4) Stock market index (random variable)

Can assign probabilities to values of a R.V.

Flip 3 fair coins Y = # of heads

Y	outcomes
0	TTT
1	HTT, THT, TTH
2	HHT, HTH, THH
3	HHH

Event $\{Y=0\} = \{TTT\}$

$\{Y=1\} = \{HTT, THT, TTH\}$

$\{Y=2\} = \cdots$

$\{Y=3\} = \cdots$

$P(Y=0) = P(\{TTT\}) = 1/8$

$P(Y=1) = P(\{HTT, THT, TTH\}) = 3/8$

$P(Y=2) = 3/8$

$P(Y=3) = 1/8$

$$\sum_{i=0}^{3} P(Y=i) = P\left(\bigcup_{i=0}^{3}\{Y=i\}\right) = P(S) = 1$$

because $\{Y=i\}$ are mutually exclusive.

In other words, can define events by conditions on the value of a random variable.

Y a R.V.

$\{Y=0\} \cup \{Y>0\} = \{Y \geq 0\}$

$\{Y \geq 0\} \cap \{Y \leq 0\} = \{Y=0\}$

$\{Y \geq 0\} \cap \{Y<0\} = \emptyset$

$\{a \leq Y \leq b\}$

Ex Urn contains 11 balls
3 white, 3 red, 5 black
+\$1 −\$1 0

We draw 3 X = amount we win or lose

All Possible values of $X = -3, -2, -1, 0, 1, 2, 3$

(An example of a discrete R.V.)

Because ONLY FINITELY MANY POSSIBLE VALUES.

$$P(X=0) = \left[\binom{5}{3} + \binom{3}{1}\binom{3}{1}\binom{5}{1}\right]\binom{11}{3}^{-1}$$

$\binom{11}{3}$ TOTAL OUTCOMES

All black; white, red, black

$$P(\text{we win money}) = P(X > 0)$$

$$= P(X = 1, 2, \text{or } 3) = P(X=1) + P(X=2) + P(X=3)$$