Review for Exam 1

HW: Ch 3 Theoretical
3.26, 3.28, 3.29 (large k limit)

Ch 4 problems:
4.1, 4.3, 4.5, 4.6

→ use the following approx

Laplace's rule of succession
$$\frac{\sum_{i=0}^{k} \left(\frac{i}{k}\right)^{n+1}}{\sum_{j=0}^{k} \left(\frac{i}{k}\right)^{n}} \approx \frac{n+1}{n+2}$$

$$\frac{1}{k}\sum_{i=0}^{k}\left(\frac{i}{k}\right)^{N} \xrightarrow[k\to\infty]{} \int_0^1 x^N\,dx = \frac{1}{N+1}$$

Ch 1 Combinatorics: Permutations / combinations and variations on such.

1: Suppose you need to answer 7 out of 10 questions. How many ways?

$$\binom{10}{7} = \frac{10!}{3!\,7!}$$

Need to take at least 3 out of first 5

$$\binom{5}{3}\binom{5}{4} + \binom{5}{4}\binom{5}{3} + \binom{5}{5}\binom{5}{2}$$

3 out of first 5 — 4 out of second 6-10

2. Consider sequences of n digits, each digit 0-9

$$10^n = \underbrace{10 \cdot 10 \cdot 10 \cdots 10}_{n}$$

No 2 consecutive digits equal?

$$10 \cdot 9^{n-1} = \underset{\text{first}}{10} \cdot \underbrace{\underset{\text{second}}{9} \cdot \underset{\text{third}}{9} \cdots 9}_{n-1}$$

If 0 appears as a digit exactly i times: $i = 0, \ldots, n$

$i = 0$: 9^n

$i = 1$: $\binom{n}{1} 9^{n-1}$

i general: $\binom{n}{i} 9^{n-i}$

3. Combinatorial Proof of

$$\binom{n}{r} = \binom{n}{n-r} \quad ?$$

Suppose have n people : split into two groups, one of size r, other of size n−r

$\binom{n}{r}$ ways choose people in first group

$\binom{n}{n-r}$ ways choose people in second group

Hence $\binom{n}{r} = \binom{n}{n-r}$ $\qquad \binom{n}{r} = \#$ of subset of n of size r

4. Lottery 40 balls numbered 1-40 in an urn, draw unordered set of 8.

$\binom{40}{8}$ possible outcomes: equally likely.

Ticket w/ 8 numbers

$$P(\text{winning}) = 1/\binom{40}{8}$$

smaller prize if you match 7

P(match exactly 7)

E_1 = match first 7 $\qquad$ # outcomes in $E_1 = 33$

E_2 = match first 7 but not eighth $\qquad \# E_2 = 32$

$$P(E_2) = \frac{32}{\binom{40}{8}}$$

$$P(\text{match exactly } 7) = \binom{8}{7} 32 \Big/ \binom{40}{8}$$

$$\binom{8}{7} = 8$$

5. E, F, G $\qquad P(E) = .22 \quad P(EF) = .11$

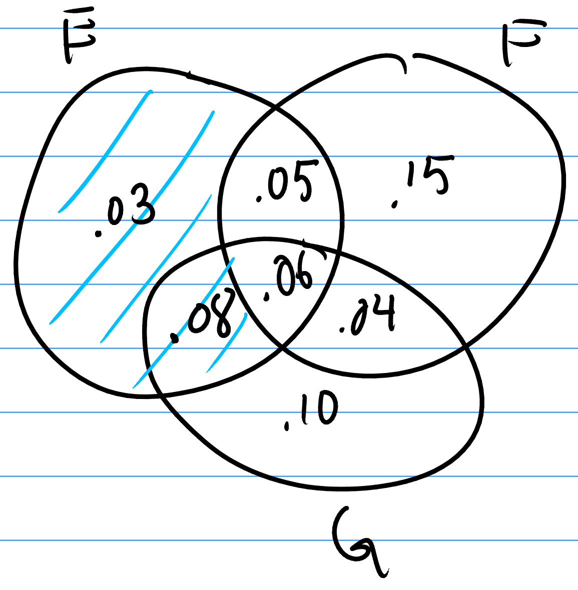
$P(F) = .30 \quad P(EG) = .14 \quad P(EFG) = .06$

$P(G) = .28 \quad P(FG) = .10$

$P(EF^c)$?

$= P(E) - P(EF)$

$= .22 - .11 = .11$



6. 2 cards are drawn from 52

$B = \{\text{both cards are aces}\}$ $\quad P(B) = \dfrac{\binom{4}{2}}{\binom{52}{2}}$

suppose we know First card is the ace of spades

$P(B \mid \text{first card is A of Spades}) = \dfrac{3}{51}$

$= \dfrac{P(B \cap \{\text{first is A of Spades}\})}{P(\{\text{first is A of spades}\})} = \dfrac{3/52\cdot 51}{1/52} = \dfrac{3}{51}$

7. You ask neighbor to water plant

$P(\text{neighbor waters}) = .9$

$P(\text{plant dies} \mid \text{watered}) = .15$

$P(\text{plant die} \mid \text{unwater}) = .8$

<u>Q1</u>: $P(\text{Plant dies}) = P(\text{die} \mid \text{water})\, P(\text{water})$
$+ P(\text{die} \mid \text{nowater})\, P(\text{nowater})$

$= (.15)(.9) + (.8)(.1)$

Q2: Given Plant dies Prob. that it was watered

$$P(water \mid die) = \frac{P(die \mid water)\, P(water)}{P(die)}$$

$$= \frac{P(die \mid water)\, P(water)}{P(die \mid water)\, P(water) + P(die \mid no\ water)\, P(no\ water)}$$