

Independent Events

Events are independent if the occurrence of one event does not affect the probability of the other event.

$$\text{In general } P(E|F) = \frac{P(EF)}{P(F)} \neq P(E)$$

But if equality holds

$$P(E|F) = P(E) \text{ or equivalently } P(EF) = P(E)P(F)$$

Then we say E and F are independent.

Examples 1 Multiple trials of given experiment

The outcomes of different trials are independent

Roll a die n times (n trials)

$$E = \{\text{even \# on first roll}\} \quad F = \{\text{get odd \# on 10th roll}\}$$

Example 2 Roll 2 dice

$$E_1 = \text{sum is 6}$$

$$E_2 = \text{sum is 7}$$

$$F = \text{first die is 4}$$

E_1 and E_2 are not independent (1st fact mut. exclusive)

(1,5) (2,4) (3,3) (4,2) (5,1)

E_1 and F

$$P(E_1) = \frac{5}{36}$$

$$P(F) = \frac{1}{6}$$

$$P(E_1, F) = \frac{1}{36}$$

$$\frac{5}{36} \cdot \frac{1}{6} \neq \frac{1}{36} \quad \underline{\text{NO}} \quad \text{not indep.}$$

E_2 and F

$$P(E_2) = \frac{6}{36} = \frac{1}{6}$$

(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)

$$P(E_2, F) = \frac{1}{36}$$

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \quad \underline{\text{Yes}} \quad \text{These are independent.}$$

Prop If E and F are independent so are E and F^c
also E^c and F
also E^c and F^c

$$E = EF \cup EF^c$$

↑
mit Exklusiv

$$P(E) = P(EF) + P(EF^c)$$

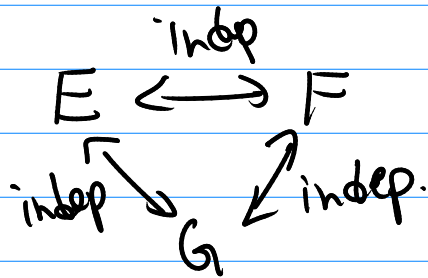
$$P(E) = P(E)P(F) + P(EF^c)$$

$$[1 - P(F)]P(E) = P(EF^c)$$

$$P(F^c)P(E) = P(EF^c) \quad \square$$

When are E , F , and G independent?

Roll 2 dice $E = \text{sum is } 7$
 $F = \text{first die is } 4$
 $G = \text{second die is } 3.$



$$P(E|FG) = 1$$

$$\frac{P(EFG)}{P(FG)} = 1$$

$$P(EFG) = P(FG)$$

$$\neq P(E)P(FG)$$

Definition

3 events E , F and G are independent (as a set of 3)

$$\text{if } P(EFG) = P(E)P(F)P(G)$$

$$\text{and } P(EF) = P(E)P(F)$$

$$P(EG) = P(E)P(G)$$

$$P(FG) = P(F)P(G)$$

n events E_1, E_2, \dots, E_n are independent

if for any subcollection of the events

$$E_{i_1}, E_{i_2}, \dots, E_{i_r}$$

we have

$$P(E_{i_1} E_{i_2} \dots E_{i_r}) = P(E_{i_1}) P(E_{i_2}) \dots P(E_{i_r})$$

i.e., the product rule holds for any subcollection of the events.

E.g. of E, F, G independent

E independent from $F \cap G, F \cup G$

NB. Mutually exclusive $EF = \emptyset \Rightarrow E$ and F are not independent
If $E \subset F \Rightarrow E$ and F are not independent

Main example Bernoulli Trials

A Bernoulli trial is an experiment w/ 2 outcomes
outcomes = {success, failure}

$$P(\{\text{success}\}) = p \quad 0 \leq p \leq 1$$

$$P(\{\text{failure}\}) = 1 - p$$

We do n trials. What is the probability of at least 1 success?

$$\{\text{at least 1 success}\}^c = \{\text{all failures}\} = F_1 F_2 \dots F_n = \bigcap_{i=1}^n F_i$$

F_i = {failure on i th trial}

$$P(F_1 F_2 \dots F_n) = \underset{\uparrow}{P(F_1) P(F_2) \dots P(F_n)} = (1-p)^n$$

Because trials are independent.

$$P(\text{at least 1 success}) = 1 - P(\text{all fail}) = 1 - (1-p)^n$$

Do n trials

Q: $P(\text{exactly } k \text{ successes})$

$P(\text{a particular sequence of } k \text{ successes and } n-k \text{ failures})$

(eg. $P(\text{SFSS}) = p^4 (1-p)^2$) \uparrow
 $= p^k (1-p)^{n-k}$

of sequence of k successes and $n-k$ $\binom{n}{k}$

$$P(\text{exactly } k \text{ success}) = \binom{n}{k} p^k (1-p)^{n-k}$$

$n=4$

$$\text{SFSS} = F_1^c F_2^c F_3^c F_4^c$$

these are independent

$$\text{FFSS} = F_1 F_2 F_3^c F_4^c$$

these are independent

these intersections are mutually exclusive