Basic Principle of Counting and Permutations

Q Suppose we flip a coin n times, what is the probability that we get heads exactly k times?

$n = 4$, $k = 2$

Total number of outcomes

HHHH	HTHH
HHHT	HTHT
HHTH	HTTH
HHTT	HTTT
THHH	TTHH
THHT	TTHT
THTH	TTTH
THTT	TTTT

16 outcomes

6 outcomes with exactly 2 heads

$$\text{probability} = \frac{6}{16}$$

Combinatorial analysis

= "the art of counting"

"Basic principle of counting"

- Suppose we perform 2 "experiments"
- The first experiment has m possible outcomes
- For each outcome of the first experiment, the second experiment has n possible outcomes

THEN, together there are $m \cdot n$ possible outcomes for the pair of experiments.

$$
m\left\{\underbrace{\begin{array}{cccc}
(1,1) & (1,2) & \cdots & (1,n) \\
(2,1) & (2,2) & & \\
(3,1) & (3,2) & \ddots & \\
\vdots & & & \\
(m,1) & & & (m,n)
\end{array}}_{n}\right.
$$

Total

$m \cdot n$

Extension:

r experiments

the ith experiment has n_i outcomes

Then all told there are

$$n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_r$$

Total outcomes.

Ex Committee of 4 students, one from each class (Fresh, soph, junior, senior)

10 freshman
22 sophmores
13 juniors
2 seniors

how many committees?

A. $10 \cdot 22 \cdot 13 \cdot 2$

Ex Suppose a computer username consists of 3 letters followed by 5 numbers

How many usernames are possible?

$$26^3 \cdot 10^5 = 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$

What if no repetitions are allowed?

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$

Ex. How many batting lineups are possible on a team with 9 players.

$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

1st batter 2nd batter 9th

Permutations = the number of ways of ordering a set of objects

(assume the objects are distinguishable)

Suppose given a set of n objects. There are n! ways to order the objects.

$n \cdot (n-1) \cdot (n-2) \cdots (2) \cdot (1) = n!$

which object is first which object is second