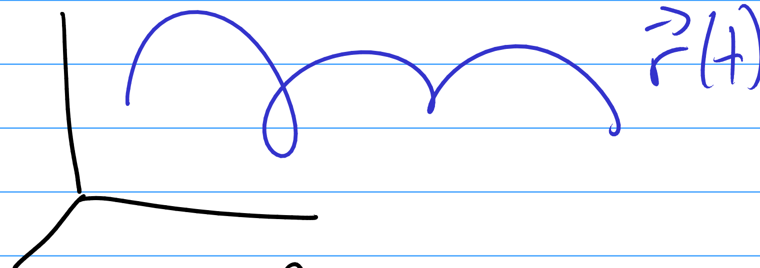
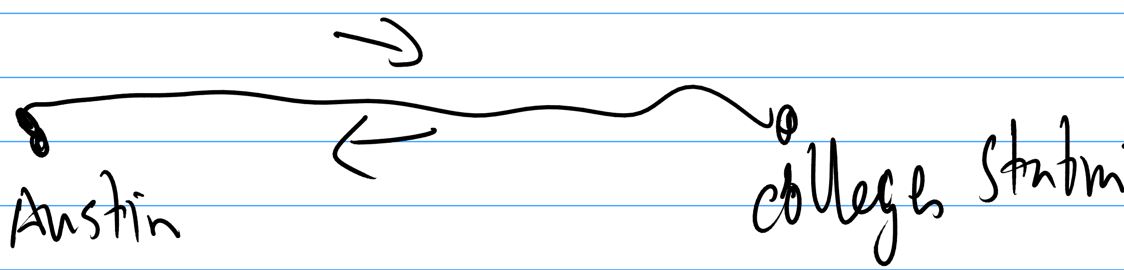
Arc length $\quad \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$



$$\text{Arclength} = \int (\text{speed})\, dt$$

$$L = \int_a^b \|\vec{r}\,'(t)\|\, dt$$

$$\|\vec{r}\,'(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}$$



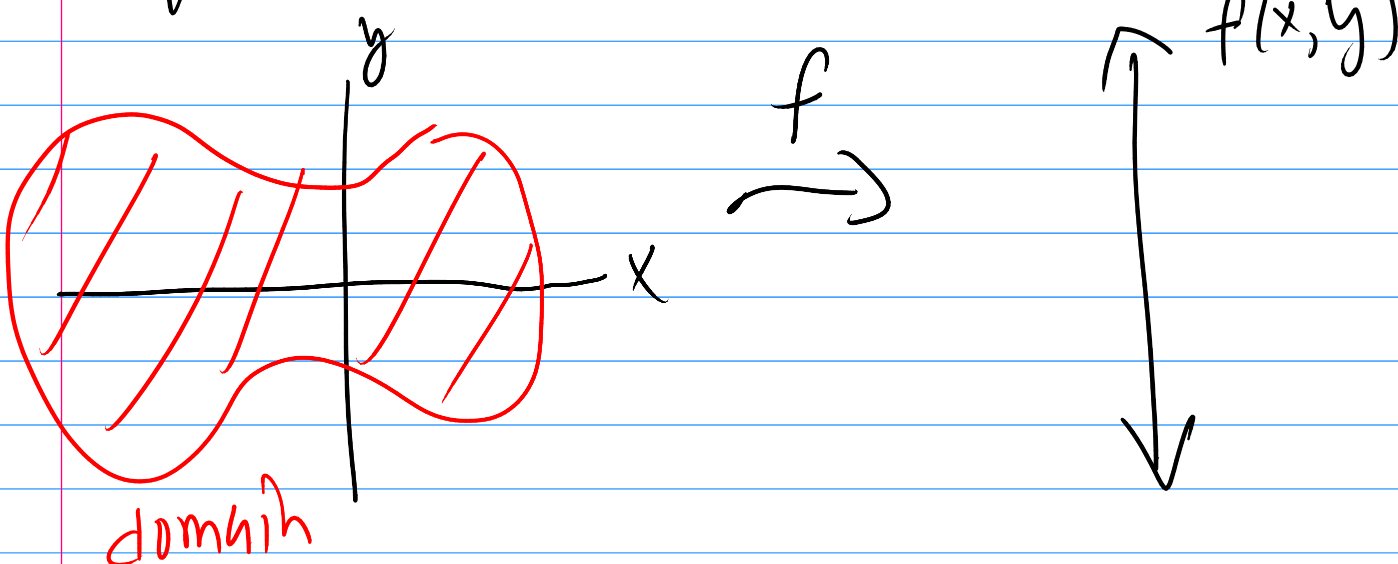
Arclength = 2 · distance

$$\int_{\text{start}}^{\text{end}} \vec{r}\,'(t)\, dt = 0$$ start and end in same place

Arclength = difference in odometer reading

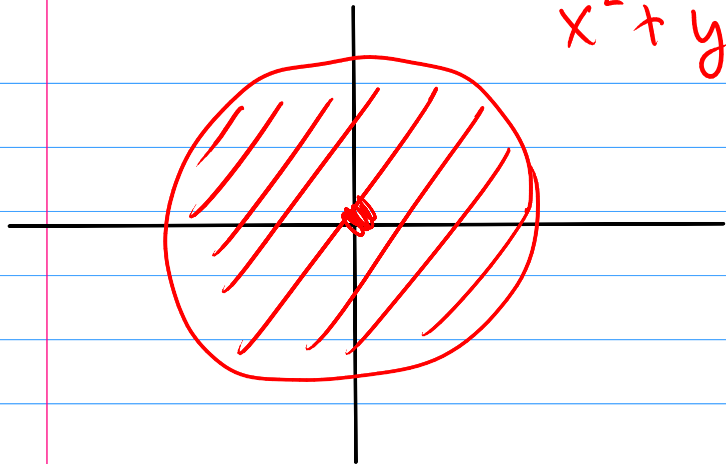
Functions of several variables
(mostly 2 variables)

$f(x,y)$ assigns a real number to each <u>pair</u> (x,y) in a subset of the xy-plane, namely the domain of f.



Q: What is the <u>natural</u> domain of

$f(x,y) = \sqrt{1-x^2-y^2}$ square root needs $\underline{1-x^2-y^2 \geq 0}$ defines the domain

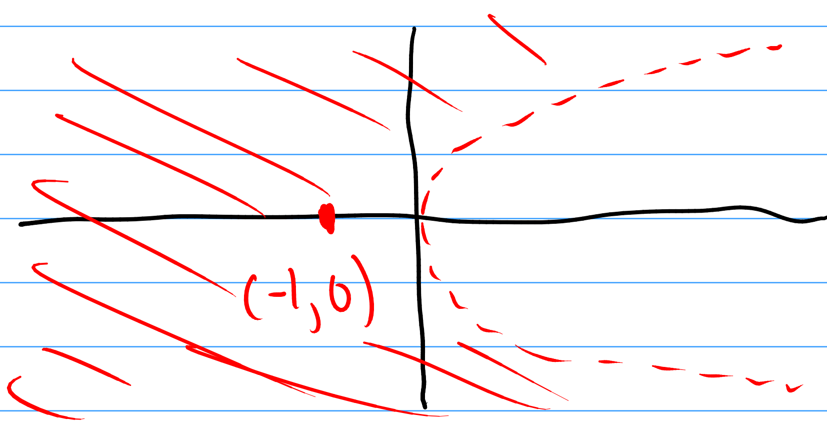


$x^2 + y^2 \le 1$

Natural domain of $f(x,y) = x \ln(y^2 - x)$

$\ln(y^2 - x)$ need $y^2 - x > 0$

$y^2 > x$



Natural domain

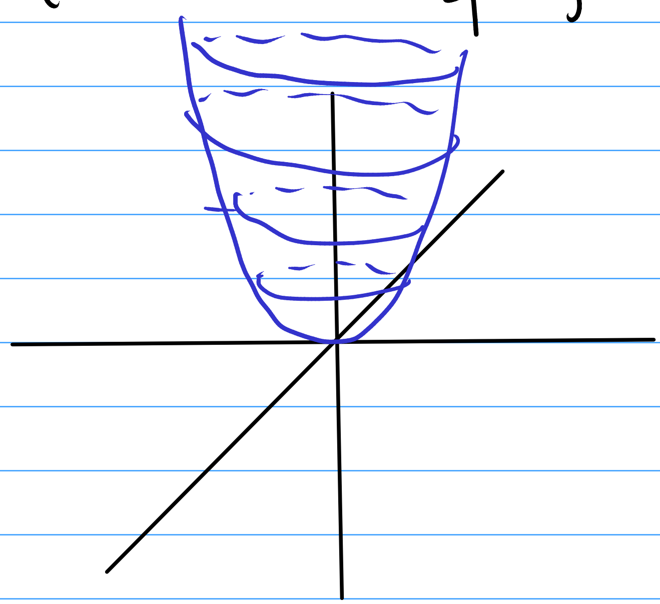
Graphs   function $f(x,y)$

"new" variable z, look at $\{z = f(x,y)\}$

which is a subset of 3-dimensional space.

Ex $f(x,y) = x^2 + \frac{y^2}{4}$ Domain is everything

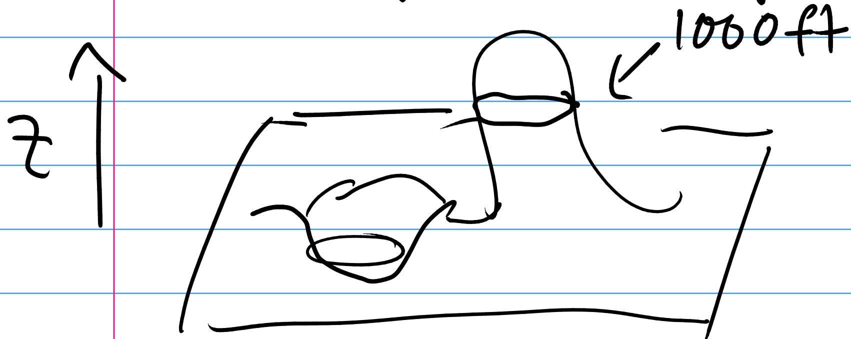
$\left\{ z = x^2 + \frac{y^2}{4} \right\}$ Elliptic Paraboloid

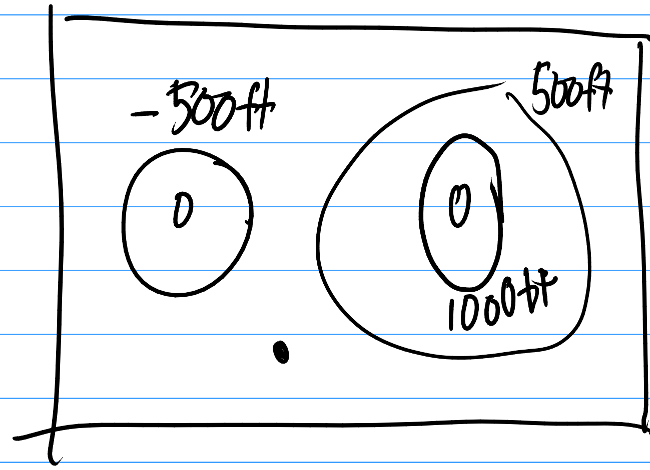


Topography: elevation $= f(x,y)$

x, y are latitude and longitude

Topographical map



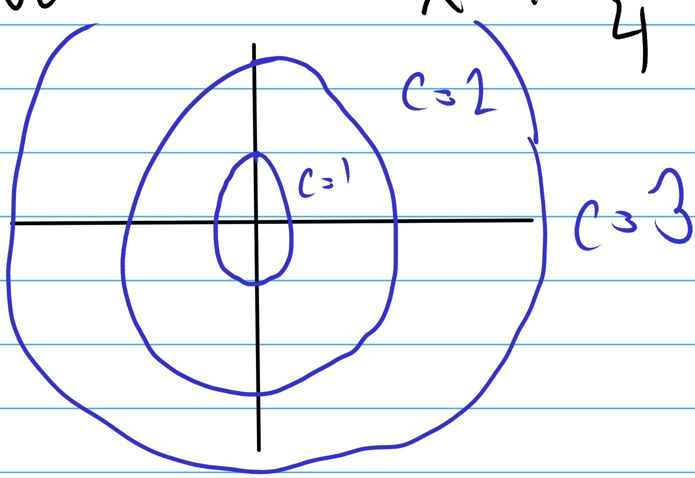


Level set, or contour plot

Level set = $\{ f(x,y) = c \}$ for some constant c (c is called the level)

$f(x,y) = x^2 + \frac{y^2}{4}$

level set $\quad x^2 + \frac{y^2}{4} = c \quad$ (ellipse



$c = -1$

$\{ x^2 + \frac{y^2}{4} = -1 \}$

is empty

3 variables: $f(x,y,z)$

<u>domain</u> is now a subset of 3-dimensional space $\quad f(x,y,z) = \sqrt{1-x^2-y^2-z^2}$

$1-x^2-y^2-z^2 \geq 0$

$1 \geq x^2+y^2+z^2$ $\quad$ solid sphere

<u>graph</u> is the solution set of $\{w = f(x,y,z)\}$ in 4-dimensions

<u>level sets</u> $\{f(x,y,z) = c\}$ for some constant c is a surface in 3-dimensions.

$\sqrt{1-x^2-y^2-z^2} = \frac{1}{2}$

$x^2+y^2+z^2 = \frac{3}{4}$ $\quad$ sphere

Limits of 2-variable functions.

$$f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}, \quad \lim_{(x,y)\to(0,0)} f(x,y)$$

TRY: Take $x \to 0$, then take $y \to 0$

$x \to 0$ get $\frac{-y^2}{y^2} = -1$, $y \to 0$ get -1

TRY Take $y \to 0$, then take $x \to 0$

$y \to 0$ get $\frac{x^2}{x^2} = 1$, $x \to 0$ get 1

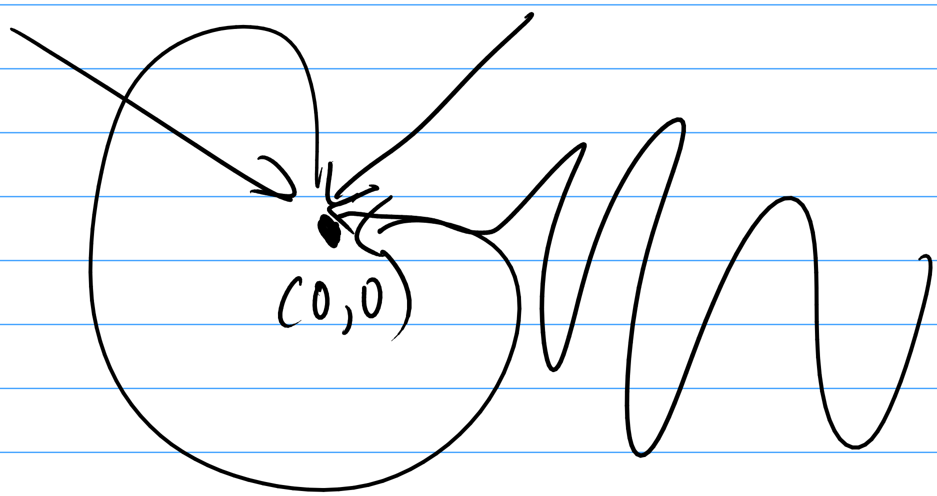
$-1 \neq 1$ get different answers.

In fact we can take the limit of $f(x,y)$ along any curve going through $(0,0)$

Parametric curve $(x(t), y(t))$

such that $(x(0), y(0)) = (0,0)$

look at $\lim_{t \to 0} f(x(t), y(t))$



Can take the limit along any of these paths

Def $\lim_{(x,y) \to (0,0)} f(x,y) = L$ means that the limit along any path exists, and is equal to a single number L.

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$ Does not exist

A function $f(x,y)$ is continuous if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$