Section 14.4, Problem 28

A batter hits a baseball 3 ft above the ground toward the center field fence, which is 10 ft high and 400 ft from home plate. The ball leaves the bat with speed 115 ft/s at an angle 50° above the horizontal. Does the ball clear the fence?

Let the x-coordinate represent the distance from home plate, while the y-coordinate denotes height above the ground. We use the foot as the unit of distance and the second as the unit of time. The intial position is then

$$\vec{r}_0 = x_0\vec{i} + y_0\vec{j} = 3\vec{j} \tag{1}$$

The initial velocity $\vec{v}_0$ is given in terms of the speed $|\vec{v}_0| = 115$ ft/s and the angle $\theta = 50°$ above the horizontal. Thus

$$\vec{v}_0 = |\vec{v}_0|(\cos\theta\vec{i} + \sin\theta\vec{j}) = 115(\cos 50°\vec{i} + \sin 50°\vec{j}). \tag{2}$$

The acceleration is $\vec{a} = -g\vec{j}$, where g is the strength of the gravitational field, measured in ft/s^2. In these units, $g \approx 32.174$.

Using the formula for motion with constant acceleration:

$$\begin{aligned}
\vec{r}(t) &= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 && (3)\\
&= 3\vec{j} + 115(\cos 50°\vec{i} + \sin 50°\vec{j})t + \frac{1}{2}(-32.174\vec{j})t^2 && (4)\\
&= (115\cos(50°)t)\vec{i} + (3 + 115\sin(50°)t - \frac{1}{2}32.174t^2)\vec{j} && (5)
\end{aligned}$$

$$\begin{aligned}
x(t) &= 115\cos(50°)t && (6)\\
y(t) &= 3 + 115\sin(50°)t - 16.087t^2 && (7)
\end{aligned}$$

The question of whether this trajectory clears the fence can be rephrased as the question of whether the ball is above the level $y = 10$ of top of the fence when the ball reaches the fence at $x = 400$.

Solving the equation $x(t) = 400$ yields the time when the ball reaches the fence:

$$t_{\text{fence}} = 400/(115\cos(50°)) \approx 5.4 \tag{8}$$

Plugging this into the equation for the height $y(t)$:

$$y(t_{\text{fence}}) = 3 + 115\sin(50°)t_{\text{fence}} - 16.087(t_{\text{fence}})^2 \approx 8.654 \tag{9}$$

Because $8.654 < 10$, the ball does not clear the fence.