Section 13.6, Problem 50

Show that the curve of intersection of the surfaces

$$x^2 + 2y^2 - z^2 + 3x = 1 \tag{1}$$

$$2x^2 + 4y^2 - 2z^2 - 5y = 0 \tag{2}$$

lies in a plane.

The curve of intersection is defined as the set of points (x, y, z) satisfing both equations (1) and (2). Therefore, such points must satisfy any combination of equations (1) and (2). In particular, we can take equation (2) minus two times equation (1):

$$\Big\{\text{equation (2)}\Big\} - 2\Big\{\text{equation (1)}\Big\} \tag{3}$$

$$\implies \quad (2x^2 + 4y^2 - 2z^2 - 5y) - 2(x^2 + 2y^2 - z^2 + 3x) = 0 - 2(1) \tag{4}$$

$$\implies \quad -5y - 6x = -2 \tag{5}$$

$$\implies \quad 6x + 5y = 2 \tag{6}$$

We conclude that every point (x, y, z) in the intersection satisfies the equation $6x + 5y = 2$. But this is the equation of a plane, and we are done.