three-dimensional coordinates.

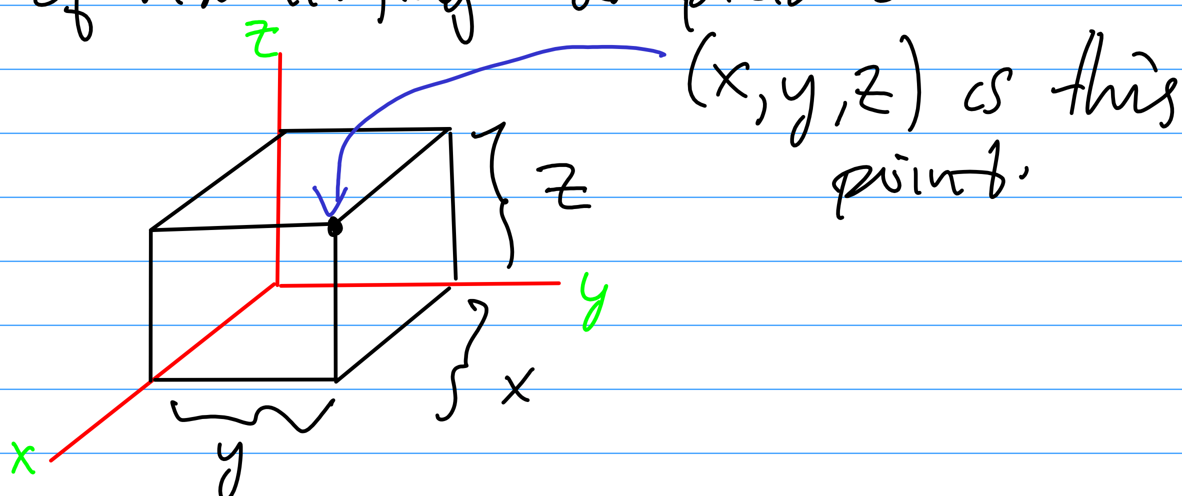
As we are quite familiar with, we need two coordinates (x, y) to represent a point in the plane.

In three-dimensional space, we need 3 coordinates (x, y, z)

Aside: This is actually one possible defn of what it means to be 3-dimensional

In fact, in higher mathematics we define the dimension of a space to be the number of coordinates we need to parametrize the points of that space.

It is important to develop the skill of visualizing 3d pictures



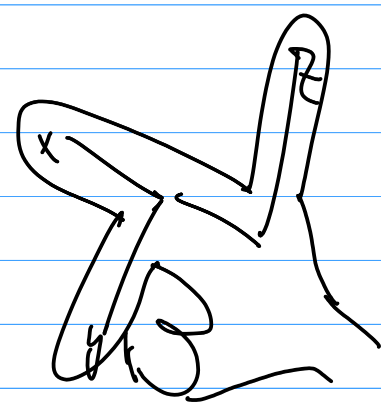
Note there are two types of 3d coordinate systems, that are equivalent in many, but not all ways

Right-handed vs. Left-handed

x-axis = index

y-axis = middle

z-axis = thumb



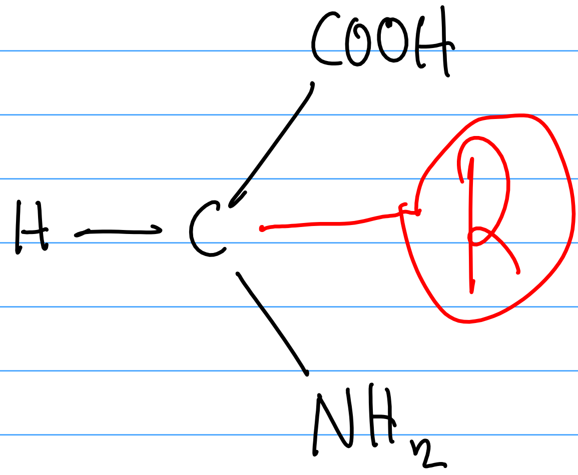
WE ALWAYS USE RIGHT-HANDED coordinate systems

[The phenomenon of "handedness" is called "Chirality".

Important in chemistry:

Chiral Molecule
Enantiomer
Stereo-isomer

left handed people have an advantage when we complete cross products.

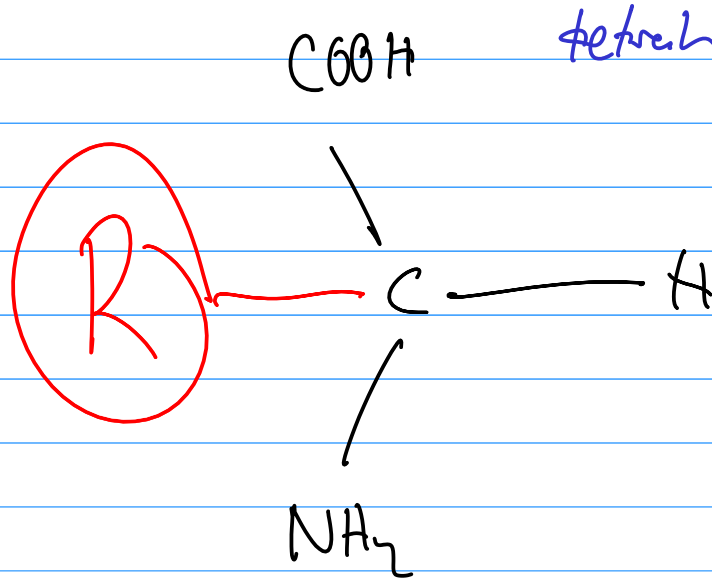


R group coming out of paper

shape is more or less tetrahedral

vs.

R group coming out of paper

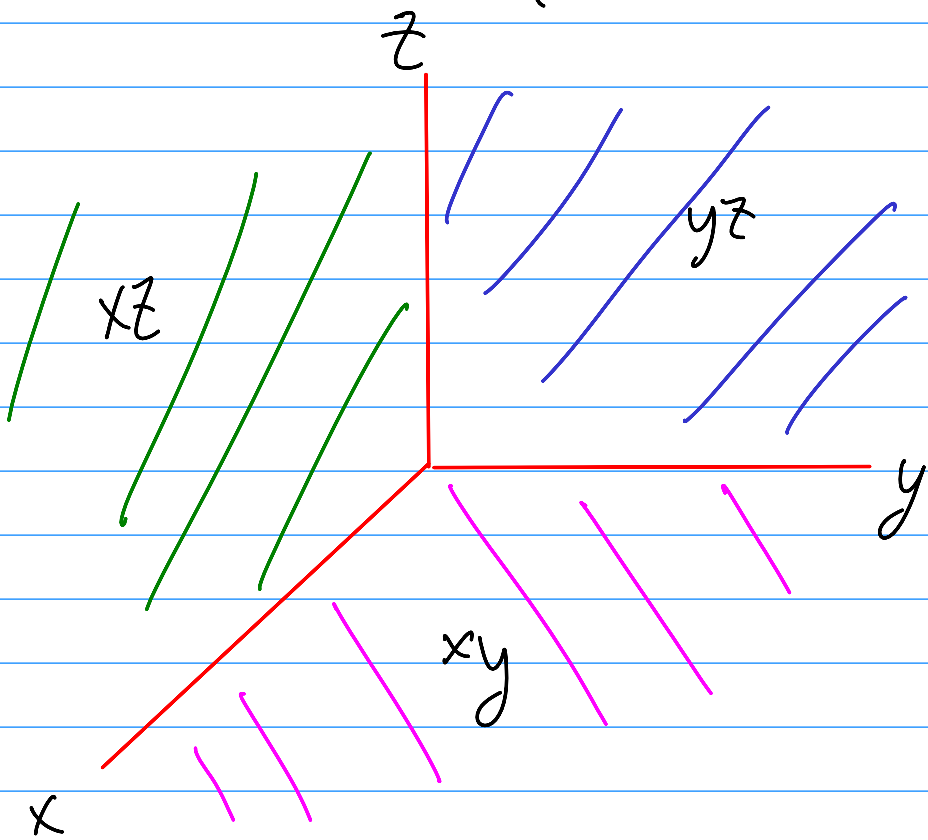


If R is a nontrivial group, these molecules are chiral
They are amino acids and they will have different biological functions.

<u>Exercise:</u> if $R = H$, the molecule is not chiral.

coordinate planes xy, yz, xz-planes are where the "third" coordinate vanishes

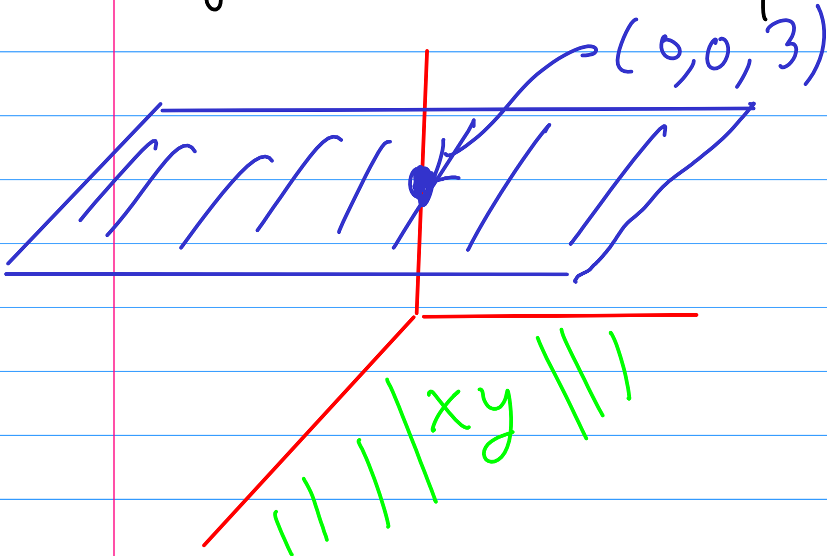
$$xy\text{-plane} = \{z=0\}$$
$$yz\text{-plane} = \{x=0\}$$
$$xz\text{-plane} = \{y=0\}$$



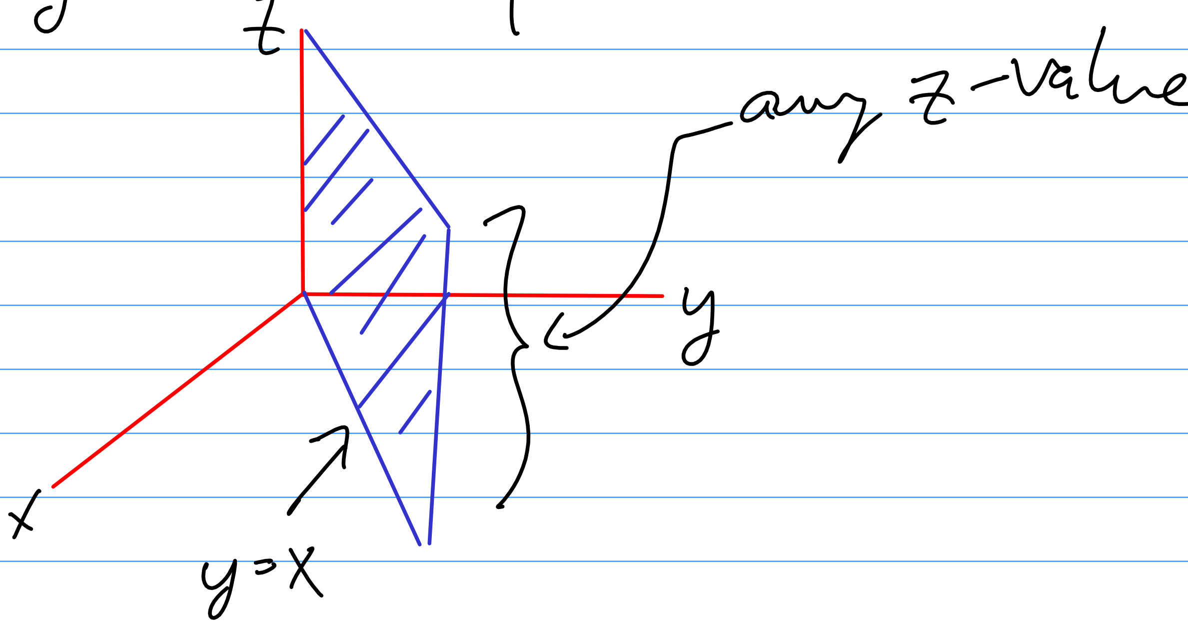
These 3-planes divide 3d space into 8 parts called octants (analogous to quadrants in 2d)

In 3d, one equation defines a surface:

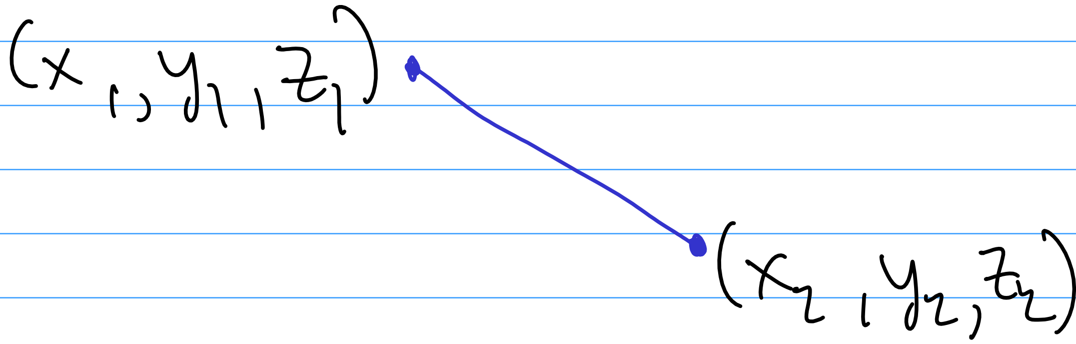
e.g. $z=3$ is a plane parallel to xy-plane



$y=x$ is a plane



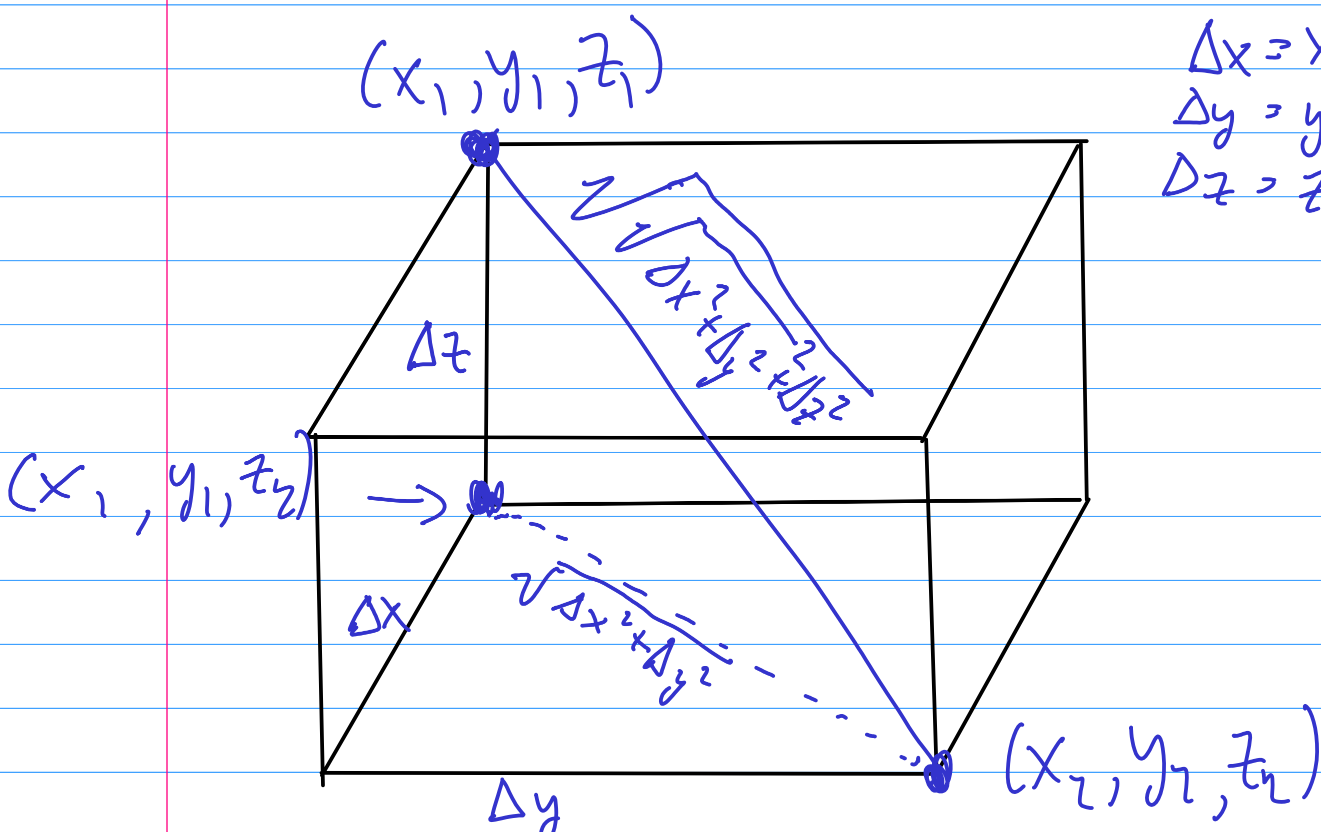
Distance formula in 3d



$$\text{distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

you can get this by applying the pythagorean theorem several times

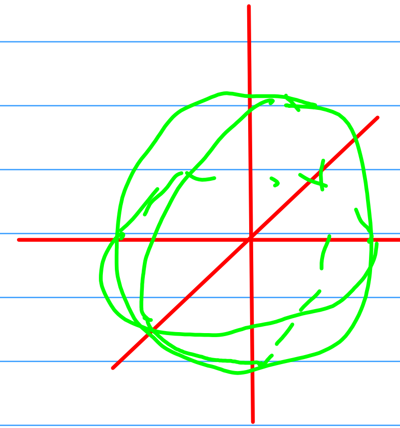
$\Delta x = x_2 - x_1$
$\Delta y = y_2 - y_1$
$\Delta z = z_2 - z_1$



Set of points equidistant from a given pt
= sphere in 3d.

Sphere centered at $(0,0,0)$

of radius r



$$\operatorname{dist}((x,y,z),(0,0,0)) = r$$

$$\sqrt{(x-0)^2+(y-0)^2+(z-0)^2} = r$$

$$x^2+y^2+z^2=r^2$$

centered at (x_0, y_0, z_0)

$$\operatorname{dist}((x,y,z),(x_0,y_0,z_0)) = r$$

$$\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} = r$$

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$$

Ex Show that $x^2+y^2+z^2-6x+4y-2z=11$ represents a sphere, find center and rad.

Complete the square

$$x^2-6x=(x-3)^2-3^2$$

$$y^2+4y=(y+2)^2-2^2$$

$$z^2-2z=(z-1)^2-1^2$$

In general

$$u^2+bu=\left(u+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2$$

$$(x-3)^2-3^2+(y+2)^2-2^2+(z-1)^2-1^2=11$$

$$(x-3)^2+(y+2)^2+(z-1)^2\underbrace{-9-4-1}_{-14}=11$$

$$(x-3)^2+(y+2)^2+(z-1)^2=25=5^2$$

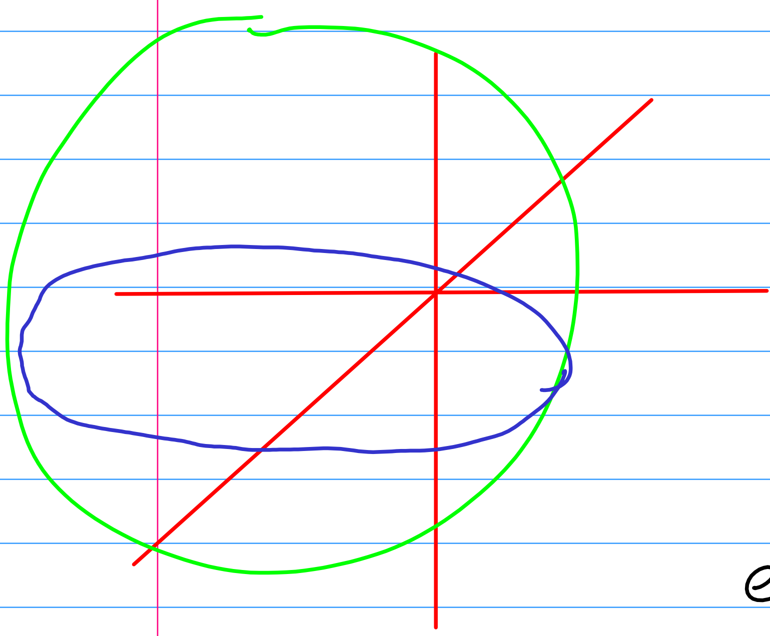
Center $=(3,-2,1)$

Radius $=5$

We have the sphere

$$(x-3)^2+(y+2)^2+(z-1)^2=5^2$$

What is the intersection of this sphere with the xy-plane $\{z=0\}$?



Intersection will be a circle.

When you intersect you're imposing both equations

$$\text{circle} = \left\{ \begin{array}{l} (x-3)^2+(y+2)^2+(z-1)^2=5^2 \\ z=0 \end{array} \right\}$$

What are the center and radius of the circle?

Need to simplify the equations.

To do this, plug $z=0$ into the first equation:

$$(x-3)^2+(y+2)^2+(0-1)^2=5^2$$

$$(x-3)^2+(y+2)^2+1=5^2=25$$

$$(x-3| + (y+2)^2 = 24 = (\sqrt{24})^2$$

center at $x=3$, $y=-2$, $z=0$

radius $=\sqrt{24}$

lies in xy-plane

Another set of equations is

$$\text{circle} = \begin{cases} (x-3)^2+(y+2)^2=24 \\ z=0 \end{cases}$$

Draw the region defined by the inequalities $\begin{Bmatrix} 1 \le x^2+y^2+z^2 \le 4 \\ z \le 0 \end{Bmatrix}$

first inequality means

$1 \le$ distance to origin ≤ 2

Second inequality means

below the xy-plane

The solid area between the spheres and below xy-plane